04-CHEM-B1, TRANSPORT PHENOMENA

DECEMBER 2016

3 hours duration

NOTES

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. The examination is an **open book exam.** One textbook of your choice with notations listed on the margins etc., but no loose notes are permitted into the exam.
3. Candidates may use any **non-communicating** calculator.
4. All problems are worth 25 points. **One problem** from **each** of sections A, B, and C must be attempted. A **fourth** problem from **any section** must also be attempted.
5. **Only the first four** questions as they appear in the answer book will be marked.
6. State all assumptions clearly.

Section A: Fluid Mechanics

A1. Water at 300 K is flowing through a smooth pipe (internal diameter = 7 cm) and $\Delta P/L$ is 125 $N/m^2.m$.

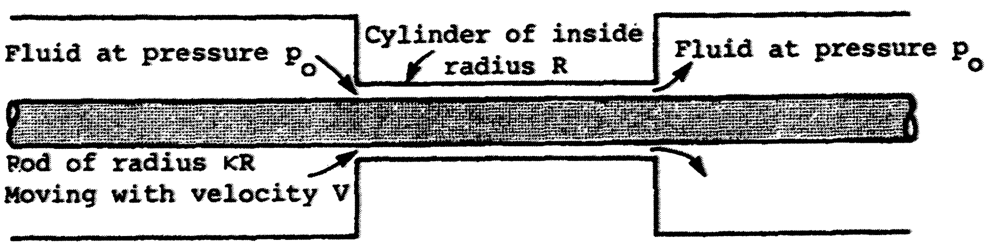
a) [15 points] Calculate the flow rate of water.

b) [10 points] Calculate the average velocity if the relative roughness of the pipe is 0.002.

<u>DATA</u>:

Density of water = 997 kg/m^3
Viscosity of water = 8.57 x 10^{-4} Pa.s

A2. Consider the flow of an incompressible fluid shown in the figure below.



Using the continuity equation and the equation for motion, obtain expressions for the velocity distribution and the volumetric flow rate.

Section B: Heat Transfer

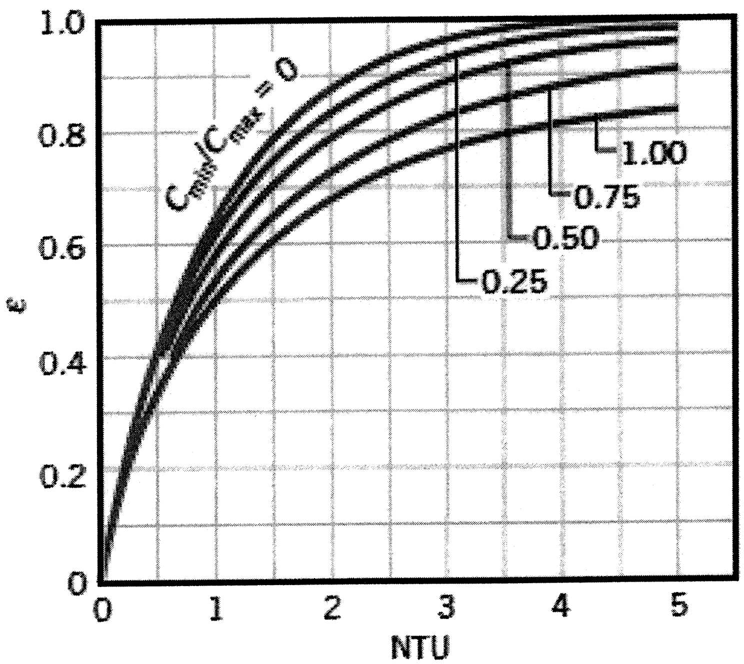
B1. Water flowing at a rate of 70 kg/min is to be heated in a counterflow double pipe heat exchanger from 40 °C to 85 °C. Oil is used as the heating agent with inlet and temperatures of 120 °C and 85 °C, respectively. For an overall heat transfer coefficient of 400 W/m^2, determine the following:

a) **[5 points]** Required heat exchanger surface area.

b) **[12 points]** Outlet water temperature, taking the same fluid inlet temperatures but with water flow rate of 50 kg/min.

c) **[8 points]** Required heat exchanger surface area for water flow rate of 50 kg/min.

<u>DATA</u>: Specific heat capacity of oil = 1.7 kJ/kg °C
Specific heat capacity of water = 4.183 kJ/kg °C



<u>Counterflow Heat Exchanger Performance</u>: **Effectiveness Factor (ε) vs. Number of Transfer Units (NTU) for Various C_{min}/C_{max} Ratios**

B2. Heat is produced by nuclear fission within a fuel rod. The rate of heat produced per unit volume (P) is a function of the radial position within the fuel rod given by the following equation:

$$\mathbf{P = P_0\,[1 + b\,(r/R_1)^2]}$$

Here R_1 is the radius of the fuel rod, b is a constant, and P_0 is the heat produced per unit volume at along the center of the fuel rod (r = 0). The fuel rod is encapped in the annular layer of cladding and cooled by heavy water at temperature T_W. The radius of the cladding is R_C, heat transfer coefficient at the cladding-heavy water interface is h_L, and the thermal conductivities of the fuel rod and cladding are k_F and k_C, respectively. Obtain an expression for the maximum temperature within the fuel rod.

Section C: Mass Transfer

C1. 0.00325 ft^3 of water spills on a surface and evaporates into still air at 74 °F and 1 atm with absolutely humidity of 0.0019 lb water/lb dry air. The saturation humidity at 74 °F is 0.0188 lb water/lb dry air. Evaporation occurs by the process of molecular diffusion through a gas film of thickness 0.19 inch. Calculate the time needed for the water spill to evaporate completely into the still air

DATA:

Diffusivity of water vapor in air at 78 °F and 1 atm = 0.258 cm^2/s
Density of water = 62.4 lb/ft^3

C2. Show that the general equation for molecular diffusion of a sphere in a stationary medium and in the absence of a chemical reaction is given by:

$$\frac{1}{D}\frac{\partial C_A}{\partial t} = \left(\frac{\partial^2 C_A}{\partial r^2}\right) + \frac{1}{r^2}\left(\frac{\partial^2 C_A}{\partial \theta^2}\right) + \frac{2}{r}\left(\frac{\partial C_A}{\partial r}\right)$$
$$+ \frac{1}{r^2 \sin^2\theta}\left(\frac{\partial^2 C_A}{\partial \varphi^2}\right) + \frac{\cot\theta}{r^2}\left(\frac{\partial C_A}{\partial \theta}\right)$$

where C_A is the concentration of the diffusing substance, D is the molecular diffusivity, t is the time, and r, θ and φ and β are spherical polar coordinates.

APPENDIX A

Summary of the Conservation Equations

Table A.1 The Continuity Equation

$$\frac{\partial \rho}{\partial t} + (\nabla \cdot \rho \vec{u}) = 0$$	(1.1)
Rectangular coordinates (*x, y, z*) $$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u_x) + \frac{\partial}{\partial y}(\rho u_y) + \frac{\partial}{\partial z}(\rho u_z) = 0$$	(1.1a)
Cylindrical coordinates (*r, θ, z*) $$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r u_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho u_\theta) + \frac{\partial}{\partial z}(\rho u_z) = 0$$	(1.1b)
Spherical coordinates (*r, θ, φ*) $$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 u_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\rho u_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(\rho u_\phi) = 0$$	(1.1c)

Table A.2 The Navier-Stokes equations for Newtonian fluids of constant ρ and μ

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\frac{1}{\rho}\nabla P + \vec{g} + \nu(\nabla^2 \vec{u})$$		(A2)
Rectangular coordinates (*x, y, z*)		
x-component	$$\frac{\partial u_x}{\partial t} + u_x\frac{\partial u_x}{\partial x} + u_y\frac{\partial u_x}{\partial y} + u_z\frac{\partial u_x}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + g_x + \nu\left(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} + \frac{\partial^2 u_x}{\partial z^2}\right)$$	(A2a)
y-component	$$\frac{\partial u_y}{\partial t} + u_x\frac{\partial u_y}{\partial x} + u_y\frac{\partial u_y}{\partial y} + u_z\frac{\partial u_y}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + g_y + \nu\left(\frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2} + \frac{\partial^2 u_y}{\partial z^2}\right)$$	(A2b)
z-component	$$\frac{\partial u_z}{\partial t} + u_x\frac{\partial u_z}{\partial x} + u_y\frac{\partial u_z}{\partial y} + u_z\frac{\partial u_z}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial z} + g_z + \nu\left(\frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_z}{\partial y^2} + \frac{\partial^2 u_z}{\partial z^2}\right)$$	(A2c)

Cylindrical coordinates (r, θ, z)

r-component

$$\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}+u_z\frac{\partial u_r}{\partial z}-\frac{u_\theta^2}{r}=-\frac{1}{\rho}\frac{\partial P}{\partial r}+g_r+\nu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(ru_r)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}+\frac{\partial^2 u_r}{\partial z^2}\right] \quad \text{(A2d)}$$

θ-component

$$\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+u_z\frac{\partial u_\theta}{\partial z}+\frac{u_r u_\theta}{r}=-\frac{1}{\rho r}\frac{\partial P}{\partial \theta}+g_\theta+\nu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(ru_\theta)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}+\frac{\partial^2 u_\theta}{\partial z^2}\right] \quad \text{(A2e)}$$

z-component

$$\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta}+u_z\frac{\partial u_z}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial z}+g_z+\nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2}+\frac{\partial^2 u_z}{\partial z^2}\right] \quad \text{(A2f)}$$

Spherical coordinates (r, θ, ϕ)

r-component

$$\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}+\left(\frac{u_\phi}{r\sin\theta}\right)\frac{\partial u_r}{\partial \phi}-\frac{u_\theta^2}{r}-\frac{u_\phi^2}{r}=-\frac{1}{\rho}\frac{\partial P}{\partial r}+g_r$$
$$+\nu\left[\frac{1}{r^2}\frac{\partial^2}{\partial r^2}\left(r^2 u_r\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial u_r}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 u_r}{\partial \phi^2}\right] \quad \text{(A2g)}$$

θ-component

$$\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+\left(\frac{u_\phi}{r\sin\theta}\right)\frac{\partial u_\theta}{\partial \phi}+\frac{u_r u_\theta}{r}-\frac{u_\phi^2}{r}\cot\theta=-\frac{1}{\rho r}\frac{\partial P}{\partial \theta}+g_\theta$$
$$+\nu\left[\begin{array}{l}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u_\theta}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(u_\theta\sin\theta\right)\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 u_\theta}{\partial \phi^2}\\ +\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}-\frac{2\cot\theta}{r^2\sin\theta}\frac{\partial u_\phi}{\partial \phi}\end{array}\right] \quad \text{(A2h)}$$

ϕ-component

$$\frac{\partial u_\phi}{\partial t}+u_r\frac{\partial u_\phi}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\phi}{\partial \theta}+\frac{u_\phi}{r\sin\theta}\frac{\partial u_\phi}{\partial \phi}+\frac{u_r u_\phi}{r}+\frac{u_\theta u_\phi}{r}\cot\theta=-\frac{1}{\rho r\sin\theta}\frac{\partial P}{\partial \phi}$$
$$+g_\varphi+\nu\left[\begin{array}{l}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u_\phi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(u_\phi\sin\theta\right)\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 u_\phi}{\partial \phi^2}\\ +\frac{2}{r^2\sin\theta}\frac{\partial u_r}{\partial \phi}+\frac{2\cot\theta}{r^2\sin\theta}\frac{\partial u_\theta}{\partial \phi}\end{array}\right] \quad \text{(A2i)}$$

Table A.3 The Energy Equation for Incompressible Media

$$\rho c_P\left[\frac{\partial T}{\partial t}+(\vec{u}\cdot\nabla)(T)\right]=\left[\nabla\cdot k\nabla T\right]+\dot{T}_G \qquad \text{(A3)}$$

Rectangular coordinates (*x*, *y*, *z*)

$$\rho c_P\left[\frac{\partial T}{\partial t}+u_x\frac{\partial T}{\partial x}+u_y\frac{\partial T}{\partial y}+u_z\frac{\partial T}{\partial z}\right]=\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{T}_G \qquad \text{(A3a)}$$

Cylindrical coordinates (*r*, *θ*, *z*)

$$\rho c_P\left[\frac{\partial T}{\partial t}+u_r\frac{\partial T}{\partial r}+\frac{u_\theta}{r}\frac{\partial T}{\partial \theta}+u_z\frac{\partial T}{\partial z}\right]=\frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(k\frac{\partial T}{\partial \theta}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{T}_G \qquad \text{(A3b)}$$

Spherical coordinates (*r*, *θ*, *ϕ*)

$$\rho c_P\left[\frac{\partial T}{\partial t}+u_r\frac{\partial T}{\partial r}+\frac{u_\theta}{r}\frac{\partial T}{\partial \theta}+\frac{u_\phi}{r\sin\theta}\frac{\partial T}{\partial \phi}\right]=$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2k\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\dot{T}_G \qquad \text{(A3c)}$$

Table A4: The continuity equation for species *A* in terms of the molar flux

$$\frac{\partial C_A}{\partial t}=-\left(\nabla\cdot\vec{N}_A\right)+\dot{R}_{A,G} \qquad \text{(4.)}$$

Rectangular coordinates (*x*, *y*, *z*)

$$\frac{\partial C_A}{\partial t}=-\left(\frac{\partial[N_A]_z}{\partial x}+\frac{\partial[N_A]_y}{\partial y}+\frac{\partial[N_A]_z}{\partial z}\right)+\dot{R}_{A,G} \qquad \text{(4a)}$$

Cylindrical coordinates (*r*, *θ*, *z*)

$$\frac{\partial C_A}{\partial t}=-\left\{\frac{1}{r}\frac{\partial}{\partial r}[rN_A]_r+\frac{1}{r}\frac{\partial}{\partial \theta}[N_A]_\theta+\frac{\partial}{\partial z}[N_A]_z\right\}+\dot{R}_{A,G} \qquad \text{(4b)}$$

Spherical coordinates (*r*, *θ*, *ϕ*)

$$\frac{\partial C_A}{\partial t}=-\left\{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2[N_A]_r\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left([N_A]_\theta\sin\theta\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}[N_A]_\phi\right\}+\dot{R}_{A,G} \qquad \text{(4c)}$$

Table A.5: The continuity equation for species *A*

$$\frac{\partial C_A}{\partial t} + (\vec{u} \cdot \nabla) C_A = D_A \nabla^2 C_A + \dot{R}_{A,G} \qquad (5)$$
Rectangular coordinates (*x*, *y*, *z*) $$\frac{\partial C_A}{\partial t} + u_x \frac{\partial C_A}{\partial x} + u_y \frac{\partial C_A}{\partial y} + u_z \frac{\partial C_A}{\partial z} = \frac{\partial}{\partial x}\left(D \frac{\partial C_A}{\partial x}\right) + \frac{\partial}{\partial y}\left(D \frac{\partial C_A}{\partial y}\right) + \frac{\partial}{\partial z}\left(D \frac{\partial C_A}{\partial z}\right) + \dot{R}_{A,G} \qquad (5a)$$
Cylindrical coordinates (*r*, *θ*, *z*) $$\frac{\partial C_A}{\partial t} + u_r \frac{\partial C_A}{\partial r} + \frac{u_\theta}{r} \frac{\partial C_A}{\partial \theta} + u_z \frac{\partial C_A}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r}\left(rD \frac{\partial C_A}{\partial r}\right) + \frac{1}{r^2} \frac{\partial}{\partial \theta}\left(D \frac{\partial C_A}{\partial \theta}\right) + \frac{\partial}{\partial z}\left(D \frac{\partial C_A}{\partial z}\right) + \dot{R}_{A,G} \qquad (5b)$$
Spherical coordinates (*r*, *θ*, *ϕ*) $$\frac{\partial C_A}{\partial t} + u_r \frac{\partial C_A}{\partial r} + \frac{u_\theta}{r} \frac{\partial C_A}{\partial \theta} + \frac{u_\phi}{r \sin\theta} \frac{\partial C_A}{\partial \phi} = \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 D \frac{\partial C_A}{\partial r}\right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta}\left(D \sin\theta \frac{\partial C_A}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial}{\partial \phi}\left(D \frac{\partial C_A}{\partial \phi}\right) + \dot{R}_{A,G} \qquad (5c)$$